Multi-User Capacity Maximization for MIMO Gaussian Broadcast Channels

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***Abstract*— In this paper we consider the problem of maximizing the multi-user capacity of Gaussian multiple-input multiple-output (MIMO) broadcast channels (BC). This problem consists in finding the optimal users' covariance matrices that maximize the multiuser capacity. These covariances represent, in the same time, the selection of users to transmit to, and their corresponding allocated power. To deal with this problem, many papers use iterative algorithms to provide the optimal solution. However, when the number of active users is high, these algorithms introduce a high order of complexity and suffer from memory drawback. Herein, we show that in a multi-user multi-antenna system, there exists a subset of active users that achieves a capacity close to the maximum, and that such iterative algorithms can be utilized considering a group of users instead of all active users. In addition, we present a new algorithm which makes a suboptimal selection of such group, referred to as the *Best Group* (BG). The proposed algorithm can be used jointly with any optimal power allocation algorithm in order to provide the covariances which maximize the multiuser capacity. Numerical results are provided and show that the BG selection is at least 5 times faster than other algorithms with a negligible reduction in the BC capacity.**

I. Introduction

Since the works by Foschini [1] and Telatar [2], a lot of efforts were dedicated to the study of systems with multiple antennas at both ends of the transmission link, commonly referred to as multiple-input multiple-output (MIMO) systems. Telatar generalized the shannon capacity of the single-input single-output (SISO) channel to the MIMO case [2]. He showed that in the single-user environment, the optimal power allocation that maximizes the mutual information of the MIMO channel is achieved through waterfilling over its eigenvalues. In the case of multi-user MIMO channels, it was proven that ordering users and using a coding which cancels the interference of subsequent users is the optimal choice to achieve the sum capacity of the network, an approach that was defined as dirty paper coding (DPC) [3]. In addition to employing DPC, there is a need for an optimal power allocation which takes into consideration the transmission power constraint(s) depending on the case of study. This problem can be distinguished for two cases: maximization of the capacity over the uplink channel, the multiple access channel (MAC) case, and over the downlink channel, the broadcast channel (BC) case.

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In the first case (MAC), many papers formulate the problem as a convex function to maximize under independent constraints over the users' transmission powers. In [4], an iterative algorithm which provides the optimal covariances that maximize the MAC capacity is proposed. The second case on the other hand is much more difficult to solve. Indeed, the BC case is a non-convex and computationally complex problem. To tackle this problem, the authors in [5] prove a duality between the MAC and BC cases, and provide a direct mapping between them. This mapping was jointly used with the iterative algorithm proposed for the MAC case [4] in order to solve the BC problem and provide the optimal covariances that maximize the downlink BC capacity [6].

In the aforementioned cases (BC and MAC), the proposed iterative algorithms take into consideration all active users in the network. Thus, for a large number of active users, such algorithms consume a large amount of computational and memory resources. In this paper, we show that there exists a small subset of users, which we call *Best Group* (BG), that achieves a BC capacity close to the maximum and that using iterative algorithms with consideration of the BG only, rather than all users, yields a significant reduction in the computational complexity and in the memory resource requirement. More specifically, we derive a simple analytical upper-bound for the capacity, based on which we propose an algorithm for selecting the BG. Iterative power allocation algorithms, such as those proposed in [4] and [6], can then be used to determine the power allocation for the selected BG.

The remainder of this paper is structured as follows: Section II presents the system model. In Section III, we provide a general overview of the MAC-BC duality and the iterative algorithm used in the BC capacity maximization. The proposed *Best Group* selection method is provided in Section IV. Performance analysis and comparisons are presented in Section V, followed by concluding remarks drawn in Section VI.

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II. System Model

We consider a broadcast channel with K users[1], $M \geq 1$ transmit antennas at the base station, and $N_k \geq 1$ receive antennas at each user equipment (UE) k. Let $\mathbf{x} \in \mathbb{C}^{M \times 1}$ be the vector of transmit signal and $\mathbf{H}_k \in \mathbb{C}^{N_k \times M}$ be the channel matrix of UE k where $\mathbf{H}_k(i,j)$ represents the channel gain from the j-th transmit antenna to the i-th receive antenna of UE k, $j = 1, \ldots, M$; $i = 1, \ldots, N_k$. The complex additive white Gaussian noise (AWGN) at UE k is represented by vector $\mathbf{n}_k \in \mathbb{C}^{N_k \times 1}$ with $\mathbf{n}_k \sim N(0, \mathbf{I})$. Let $\mathbf{y}_k \in \mathbb{C}^{N_k \times 1}$ be the received signal at UE k, the input-output relationship of the system can then be represented as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k \qquad k = 1, \ldots, K. \tag{1}$$

The base station transmitter is subject to an average power constraint P, which implies that $\operatorname{tr}(\mathbf{\Sigma_x}) \leq P$ where $\operatorname{tr}(\cdot)$ denotes the trace function and $\mathbf{\Sigma_x} \triangleq \mathbb{E}[\mathbf{x}\mathbf{x}^\dagger]$ is the covariance matrix of the input signal $\mathbf{x}$ where $\mathbb{E}$ stands for mathematical expectation. For $k = 1, \ldots, K$, we define the channel matrix

$$\overline{\mathbf{H}}_k \triangleq [\mathbf{H}_1^\dagger \ldots \mathbf{H}_{k-1}^\dagger \mathbf{H}_{k+1}^\dagger \ldots \mathbf{H}_K^\dagger], \tag{2}$$

where $()^\dagger$ denotes the conjugate transpose (i.e. Hermitian) operator. Finally, we assume that the channel matrices $\{\mathbf{H}_k\}_{k=1}^K$ are perfectly known at the base station.

III. Duality and Broadcast Channel Capacity Maximization

As previously mentioned, in the broadcast channel (BC) case, formulation of the dirty paper region is rather complicated. Indeed, the MIMO BC capacity achieved by dirty paper coding (DPC) [3] can be written as,

$$\begin{aligned}
\mathcal{C}_{\mathrm{BC}}(\mathbf{H}_1, \ldots, \mathbf{H}_K, P) = \\
\max_{\{\mathbf{\Sigma}_k : \mathbf{\Sigma}_k \succeq 0, \sum_{k=1}^K \operatorname{tr}(\mathbf{\Sigma}_k) \leq P\}} \quad & \log \left| \mathbf{I} + \mathbf{H}_1 \mathbf{\Sigma}_1 \mathbf{H}_1^\dagger \right| \\
+ \quad & \log \frac{\left| \mathbf{I} + \mathbf{H}_2 (\mathbf{\Sigma}_1 + \mathbf{\Sigma}_2) \mathbf{H}_2^\dagger \right|}{\left| \mathbf{I} + \mathbf{H}_2 \mathbf{\Sigma}_1 \mathbf{H}_2^\dagger \right|} \\
+ \cdots + \quad & \log \frac{\left| \mathbf{I} + \mathbf{H}_K \sum_{k=1}^K \mathbf{\Sigma}_k \mathbf{H}_K^\dagger \right|}{\left| \mathbf{I} + \mathbf{H}_K \sum_{k=1}^{K-1} \mathbf{\Sigma}_k \mathbf{H}_K^\dagger \right|},
\end{aligned} \tag{3}$$

where notation $\mathbf{Q} \succeq 0$ is used to indicate that $\mathbf{Q}$ is positive semi-definite, and $|\cdot|$ denotes the determinant of a matrix.

It was shown in [5] that the maximum capacity achieved by DPC in the MIMO BC case equals the capacity of the dual MIMO MAC case, which is a convex problem that can be solved more easily than the former case. The dual uplink is formed by considering the transmitter to be an M-antenna receiver and assuming each UE k to be a N_k-antenna transmitter. The received signal in the dual MAC case is given by [5],

$$\mathbf{y}_{\mathrm{MAC}} = \sum_{k=1}^K \mathbf{H}_k^\dagger \mathbf{x}_k + \mathbf{n}, \tag{4}$$

where $\mathbf{n} \sim N(0, \mathbf{I})$.

Using the MAC-BC duality, the capacity (3) of the MIMO BC can be reformulated as,

$$\begin{aligned}
&\mathcal{C}_{\mathrm{BC}}(\mathbf{H_1}, \ldots, \mathbf{H}_K, P) \\
&= \max_{\{\mathbf{Q}_k : \mathbf{Q}_k \succeq 0, \sum_{k=1}^K \operatorname{tr}(\mathbf{Q}_k) \leq P\}} \log \left| \mathbf{I} + \sum_{k=1}^K \mathbf{H}_k^\dagger \mathbf{Q}_k \mathbf{H}_k \right|,
\end{aligned} \tag{5}$$

where $\{\mathbf{Q}_k\}_{k=1}^K$ are the uplink covariance matrices of the K UEs. The downlink covariances $\{\mathbf{\Sigma}_k\}_{k=1}^K$ can then be deduced directly from $\{\mathbf{Q}_k\}_{k=1}^K$ using the MAC-BC mapping provided in [5].

Using the MAC-BC duality and following the iterative algorithm used to solve the MIMO MAC case [4], the authors in [6] provided a modified algorithm adapted to the MIMO BC case. This algorithm uses an iterative DPC, which we refer to hereafter as IDPC. In summary, the n^{th} iteration of the IDPC algorithm is formulated as follows [6]:

1) Generation of the effective channels $\{\mathbf{G}_k^{(n)}\}_{k=1}^K$ according to:

$$\mathbf{G}_k^{(n)} = \mathbf{H}_k \left(\mathbf{I} + \sum_{j \neq k} \mathbf{H}_j^\dagger \mathbf{Q}_j^{(n-1)} \mathbf{H}_j \right)^{-1/2}. \tag{6}$$

2) Treating these effective channels as parallel non-interfering channels, obtain the new covariance matrices $\{\mathbf{Q}_k^{(n)}\}_{k=1}^K$ using waterfilling with total power P:

$$\begin{aligned}
&\mathbf{Q}_k^{(n)} = \\
&\max_{\left\{\mathbf{Q}_k : \mathbf{Q}_k \succeq 0, \sum_{k=1}^K \operatorname{tr}(\mathbf{Q}_k) \leq P\right\}} \sum_{k=1}^K \log \left| \mathbf{I} + \left(\mathbf{G}_k^{(n)} \right)^\dagger \mathbf{Q}_k \mathbf{G}_k^{(n)} \right|.
\end{aligned} \tag{7}$$

As aforementioned, maximizing the MIMO BC capacity is equivalent to finding the uplink covariances $\{\mathbf{Q}_k\}_{k=1}^K$ which maximize the MAC capacity. These covariance matrices define, at the same time, (i) the selected UEs to transmit to, and (ii) the power allocation over each channel (i,j) between the j-th transmit antenna at the base station, and the i-th receive antenna at one UE.

In the following, we show that there exists a small group of UEs that maximizes the MIMO BC capacity, even if there are many UEs for which the base station can transmit to. In general, the number of transmit antennas is much smaller than the number of active users, yielding a limitation on the number of orthogonal channels between the transmit antennas and the UEs' antennas. In order to reduce the computational complexity and the memory resources required in solving the

[1]The terms "user" and "user equipment" (UE) are used interchangeably.

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BC problem (5), which grow as the number of UEs, K, gets higher, we propose to make the selection of UEs and their corresponding power allocation, at two different steps as will be explained in the next section. The subset of selected UEs will be denoted by *Best Group* (BG).

IV. The Best Group of Users Approach

A. Best-Group Concept

It is well known that, in order to maximize the capacity of the single-transmit antenna ($M = 1$)/multiple receive antennas (SIMO) broadcast channel, selecting the UE with the best channel quality as the user to transmit to, constitutes the best choice among the active users, while for the MIMO BC, this is does not hold true. On the other hand, previous works dealing with the MIMO MAC case, show that optimality is achieved by combining spatial diversity and user multiplexing [7]. Using the MAC-BC duality, we may conclude that this is also true for the MIMO BC case. In particular, we show herein that there exists a subset of UEs, which achieves a capacity almost equal to the maximum, independently of the power allocation used for each UE.

To illustrate this, consider $K = 20$ UEs with independent fading channels, the same number of receive antennas, N_{R}, and a total power constraint of $P = 1$. Figure 1 presents the average BG size, denoted by N_{BG}, and the confidence interval with a level equal to 95% required to achieve 99% of the maximal MIMO BC capacity (i.e. using IDPC), as a function of the number of transmit antennas at the base station, M, and for different values of the number of UE antennas, $N_R = 1, 2$ or 3. This figure shows the fact that for one transmit antenna ($M = 1$), the optimal choice consists in selecting one UE at each transmission interval. For other values of M, the BG size N_{BG} is small compared to the number of active users, $K = 20$. This suggests that a selection of the BG prior to determining the optimal power allocation, allows to reduce the computational complexity, a reduction that can be considerable when K is large. Hence, in the IDPC iterative algorithm, presented above, instead of considering all K users, we can just consider the UEs that belong to the BG. Accordingly, the maximization problem in (5) can be reformulated as,

$$\widetilde{\mathcal{C}}_{\mathrm{BC}}(\mathbf{H}_1, \ldots, \mathbf{H}_K, P) = \max_{\{\mathbf{Q}_k : \mathbf{Q}_k \succeq 0, \sum_{k\in \mathrm{BG}} \mathrm{tr}(\mathbf{Q}_k) \le P\}} \log \left| \mathbf{I} + \sum_{k \in \mathrm{BG}} \mathbf{H}_k^\dagger \mathbf{Q}_k \mathbf{H}_k \right| . \quad (8)$$

Now, there are two remaining open questions, which can be summarized as follows: (i) What is the BG size which provides a capacity close to the maximum MIMO BC capacity, say at least 95% of the latter? and (ii) How to select the BG using simple procedure?. In the following, we consider that the number of UEs K is larger than the number of transmit antennas M, which is the more common practical operating scenario, and provide our solutions to these problems.

B. Best-Group Selection Algorithm

In order to simplify the selection procedure of the BG, we present a selection algorithm which provides the set of UEs (i.e., BG) that achieves a MIMO BC capacity close to the maximum based on an upper-bound for this capacity. In general, the group of users which maximizes the upper-bound does not necessarily represent the group which maximizes the BC capacity, which can yield to sub-optimality in the selection procedure. Nevertheless, the resulting capacity reduction is not significant as we will shortly show.

The expression in (5) can be written as,

$$\mathcal{C}_{\mathrm{BC}}(\overline{\mathbf{H}}, P) = \max_{\{\mathbf{Q} \in \mathcal{M}_D : \mathbf{Q} \succeq 0, \mathrm{tr}(\mathbf{Q}) \le P\}} \log \left| \mathbf{I} + \overline{\mathbf{H}} \mathbf{Q} \overline{\mathbf{H}}^\dagger \right| , \quad (9)$$

where $\overline{\mathbf{H}} \triangleq [\mathbf{H}_1^\dagger \ldots \mathbf{H}_K^\dagger]$, $\mathcal{M}_D$ denotes the ensemble of block diagonal matrices of size $N \times N$ where $N \triangleq \sum_{k=1}^K N_k$, and $\mathbf{Q}$ is a block-diagonal covariance matrix given by

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{Q}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{Q}_K \end{bmatrix} .$$

To simplify the problem at hand and based on the results shown in Fig. 1, an acceptable BG size, N_{BG}, can be chosen equal to the rank of matrix $\overline{\mathbf{H}}$, i.e., $L = \min(M, N)$.

Since $\mathcal{M}_D \subset \mathbb{C}^{N \times N}$, an upper-bound for $\mathcal{C}_{\mathrm{BC}}(\overline{\mathbf{H}}, P)$ can be obtained by performing the maximization in (9) over $\mathbb{C}^{N \times N}$ instead of $\mathcal{M}_D$, thus,

$$\mathcal{C}_{\mathrm{BC}}(\overline{\mathbf{H}}, P) \le \mathcal{C}_{\mathrm{up}}(\overline{\mathbf{H}}, P), \quad (10)$$

where

$$\mathcal{C}_{\mathrm{up}}(\overline{\mathbf{H}}, P) = \max_{\{\widetilde{\mathbf{Q}} \in \mathbb{C}^{N \times N} : \widetilde{\mathbf{Q}} \succeq 0, \mathrm{tr}(\widetilde{\mathbf{Q}}) \le P\}} \log \left| \mathbf{I} + \overline{\mathbf{H}} \widetilde{\mathbf{Q}} \overline{\mathbf{H}}^\dagger \right| , \quad (11)$$

which is equal to the MIMO BC capacity of a unique UE with channel matrix $\overline{\mathbf{H}}$, and limited by a power constraint equal to P. The well-known solution to this problem is the waterfilling technique [2], with the covariance matrix $\widetilde{\mathbf{Q}}$ which maximizes the capacity of a diagonal form. Hence,

$$\mathcal{C}_{\mathrm{up}}(\overline{\mathbf{H}}, P) = \sum_{j=1}^{M} [\log(\mu \lambda_j)]^+ , \quad (12)$$

where $[x]^+ \triangleq \max(x, 0)$, λ_j denotes the j^{th} eigenvalue of matrix $\overline{\mathbf{H}}^\dagger \overline{\mathbf{H}}$ with $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_M$, and μ is the water level satisfying $P = \sum_{j=1}^M [\mu - \lambda_j^{-1}]^+$.

Selection of the BG is performed by computing the equivalent upper-bound capacity for each UE k when the latter is excluded from the set of active users, i.e., $\mathcal{C}_{\mathrm{up}}(\overline{\mathbf{H}}_k, P)$, taking into consideration the channel matrix $\overline{\mathbf{H}}_k$ (2) instead of $\overline{\mathbf{H}}$. The BG is the set of UEs that provides the M smallest values of $\left\{\mathcal{C}_{\mathrm{up}}(\overline{\mathbf{H}}_{\mathbf{k}}, P)\right\}_{k=1}^K$. Indeed, this set of UEs has the major effect on decreasing the upper-bound capacity, which means that it is, with high probability, the set of UEs that maximizes the BC capacity. The BG selection procedure is detailed in Algorithm

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(1). This selection procedure is followed by power allocation by means of the IDPC algorithm, considering the selected users included in the BG. The combined processes, i.e., BG selection followed by the IDPC power allocation algorithm, is referred to as *BG-IDPC*.

Algorithm 1 The Best Group Selection

INPUT: – Channel matrices $\{\mathbf{H}_k\}_{k=1}^{K}$,
– Power budget P.
OUTPUT: The Best Group of users (BG).
PROCEDURE:
for $k = 1$ to K **do**
Compute the eigenvalues of $\overline{\mathbf{H}}_k^{\dagger}\overline{\mathbf{H}}_k$, $\{\lambda_{k,j}\}_{j=1}^{M}$;
Compute the water level μ such that

$$P = \sum_{j=1}^{M}[\mu - \lambda_{k,j}^{-1}]^{+};$$

Compute the upper-bound capacity

$$\mathcal{C}_{\mathrm{up}}(\overline{\mathbf{H}}_k, P) = \sum_{j=1}^{M}\left[\log(\mu\lambda_{k,j})\right]^{+};$$

end for
Sort the upper-bound capacities $\{\mathcal{C}_{\mathrm{up}}(\overline{\mathbf{H}}_k, P)\}_{k=1}^{K}$ in increasing order;
Generate the Best Group by collecting the UEs' indices that provide the M smallest values of $\{\mathcal{C}_{\mathrm{up}}(\overline{\mathbf{H}}_k, P)\}_{k=1}^{K}$.

In order to keep the capacity reduction at a target level, a dynamic updating of the BG size value N_{BG} can be implemented. The proposed method consists of starting with $N_{\mathrm{BG}} = \min(N, M)$, and incrementing N_{BG} as needed in order to keep the achievable capacity at an acceptable level. The increase of the BG set by one UE at a time if necessary requires a reallocation of the power between previously selected UEs. However, this reallocation does not introduce a significant change in their corresponding power allocation. In fact, each new added UE has the weakest channel compared to the channels of UEs previously selected in the BG, and consequently its corresponding power allocation is too small to really affect the previous power allocation. Thus, the power allocation step considering the updated BG size, will quickly converge to the optimal power values, and as such, the additional complexity will be negligible.

Here it is important to mention that the complexity of the BG-IDPC algorithm is given by $\mathcal{O}(K.N.M^2)$ and that the one corresponding to the original IDPC algorithm, which considers all active users, is given by $\mathcal{O}(T.K.N.M^2)$ where T is the number of iterations needed for the algorithm to converge to the optimal solution. Hence, the complexity of the BG-IDPC algorithm is at least T times less than that of the IDPC algorithm.

V. Performance Evaluation

Hereafter, we present simulation results using the BG-IDPC algorithm and an evaluation of its performance in comparison to the IDPC iterative algorithm presented in Section III. Comparisons are made in terms of the execution time and the achievable capacity between the BG-IDPC, that is the BG algorithm used jointly with the IDPC iterative algorithm, and the IDPC algorithm used without the pre-selection of the BG.

Considering $K = 20$ UEs and a MIMO configuration with $M = 4$ transmit antennas and $N_R = 4$ receive antennas for all UEs, Fig. 2 plots the execution time for different channel realizations $\overline{\mathbf{H}}$. Simulation results show that the BG-IDPC procedure is at least 5 times faster than the IDPC one. The effect of varying the number of active users, K, on the execution time is shown in Fig. 3, where both schemes are considered. As shown in [6], the complexity of the IDPC algorithm is a linear function of the number of UEs K. The same observation can be made for the BG-IDPC method except that the latter is at least 5 times faster than IDPC. Thus, the pre-selection of the BG prior to using an iterative algorithm for the power allocation significantly reduces the execution time compared to the IDPC algorithm at the cost of a negligible reduction in capacity.

To quantify the reduction in capacity, define $\Delta\mathcal{C}_{\mathrm{BC}} = \frac{\mathbb{E}[\mathcal{C}_{\mathrm{BC}}]-\mathbb{E}[\widetilde{\mathcal{C}}_{\mathrm{BC}}]}{\mathbb{E}[\mathcal{C}_{\mathrm{BC}}]}$. Figure 4 shows the average proportional capacity reduction $\Delta\mathcal{C}_{\mathrm{BC}}$, in presence of $K = 20$ UEs for different MIMO configurations and for two values of the BG size ($N_{\mathrm{BG}} = L$ and $N_{\mathrm{BG}} = L + 1$). We observe that for $N_{\mathrm{BG}} = L$, the capacity reduction is less than 5% and decreases as the number of transmit antennas M increases, while when increasing the BG size by one ($N_{\mathrm{BG}} = L + 1$), the capacity achieves 98% of its maximal value. As expected for the single-input single-output (SISO) scheme, there is no reduction in capacity since the UE with the best channel is the one selected.

Figure 5 presents results pertaining to the dynamic updating of the N_{BG} value in order not to exceed 5% reduction in the instantaneous MIMO BC capacity when using BG-IDPC compared to that achieved using IDPC. The average value of N_{BG} considering this constraint is approximatively equal to $\min(N, M) + 1 = 5$, which is small compared to the number of active users, $K = 20$. Furthermore, the illustration in Fig. 6 of the variations of the reduction in capacity, defined as $\delta\mathcal{C}_{\mathrm{BC}} = \frac{\mathcal{C}_{\mathrm{BC}}-\widetilde{\mathcal{C}}_{\mathrm{BC}}}{\mathcal{C}_{\mathrm{BC}}}$, shows that the latter never exceeds the 5% limit.

The presented results show that the pre-selection of the BG is an efficient technique for reducing the complexity of iterative algorithms, such as the IDPC, or other non-iterative algorithms which are time-consuming and have memory drawbacks. Indeed, even with the use of a sub-optimal selection algorithm, the BG-IDPC provides more than 95% of the maximal MIMO BC capacity, and is at least 5 faster than the IDPC algorithm that considers all active users. Moreover, using our proposed BG-IDPC algorithm with a dynamic updating of the BG size, N_{BG}, we can satisfy any constraint on the acceptable capacity reduction.

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VI. Conclusions

This paper studied the problem of maximizing the MIMO Broadcast Channel (BC) multiuser capacity under total power constraint. The solution to achieve the optimal capacity is the use of iterative algorithms such as the iterative dirty paper coding (IDPC) method. However, such algorithms are time and memory consuming. We showed that there exists a subset of users in the network which achieves a capacity close to the maximum, and that a selection of such group, denoted as Best Group (BG), prior to performing the power allocation considerably reduces the complexity of the IDPC algorithm. In particular, based on an upper-bound for the capacity, we presented a simple selection method for the BG which, when used jointly with the IDPC algorithm, yields at least 95% of the maximum BC capacity and is at least 5 times faster than the use of the IDPC algorithm with consideration of all active users in the network. Furthermore, a dynamic updating of the BG size was proposed so as to keep the achievable capacity at an acceptable level. Future work includes the presentation of an optimal dynamic selection method for the BG with a fast power allocation method.

References

[1] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," in *Wireless Personal Communications*, vol. 6, Mar. 1998, pp. 311–335.

[2] E. Telatar, "Capacity of multi-antenna gaussian channels," in *Europ. Trans. on Telecomm. (ETT)*, vol. 10, Nov. 1999, pp. 585–596.

[3] M. Costa, "Writing on dirty paper," *IEEE Trans. Inform. Theory*, vol. 29, pp. 439–441, May 1983.

[4] Y. Wei, R. Wonjong, S. Boyd, and J. Cioffi, "Iterative water-filling for gaussian vector multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 50, pp. 145–152, Jan. 2004.

[5] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of gaussian MIMO broadcast channels," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2658–2668, Oct. 2003.

[6] N. Jindal, R. Wonjong, S. Vishwanath, S. Jafar, and A. Goldsmith, "Sum power iterative water-filling for multi-antenna gaussian broadcast channels," *IEEE Trans. Inform. Theory*, vol. 51, pp. 1570–1580, Apr. 2005.

[7] D. Tse, P. Viswanath, and Z. Lizhong, "Diversity-multiplexing tradeoff in multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 50, pp. 1859–1874, Sept. 2004.

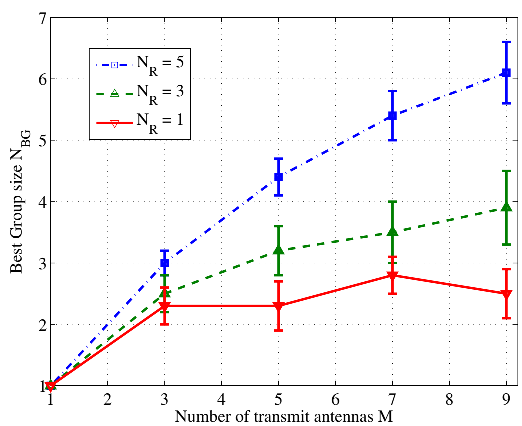

Fig. 1. Average Best Group size as a function of the number of transmit antennas M at the base station, for different values of receive antennas N_R, with $K = 20$ and $P = 1$.

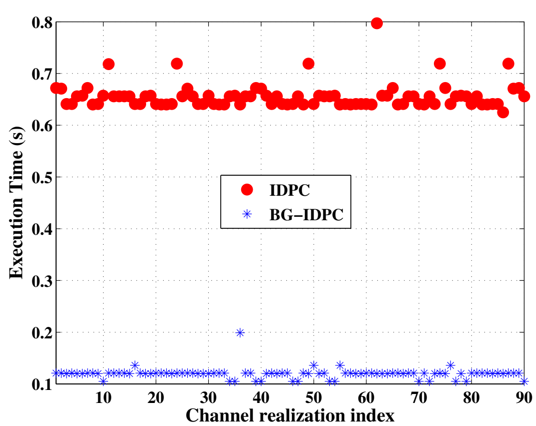

Fig. 2. Comparison of the execution time between the use of IDPC and the BG-IDPC algorithms for different channel realizations $\overline{\mathbf{H}}$, with $K = 20$ and a 4×4 MIMO configuration.

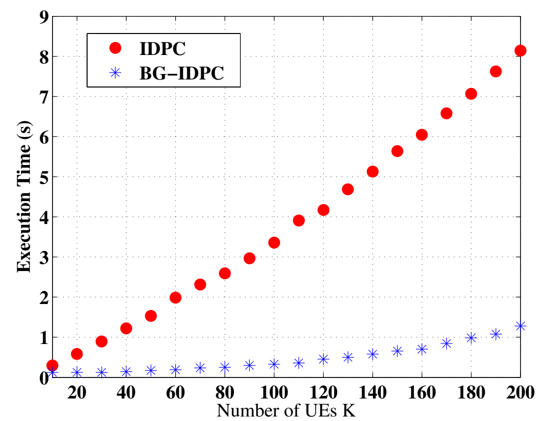

Fig. 3. Execution time evolution for IDPC and BG-IDPC as a function of the number of UEs K in a 4×4 MIMO configuration.

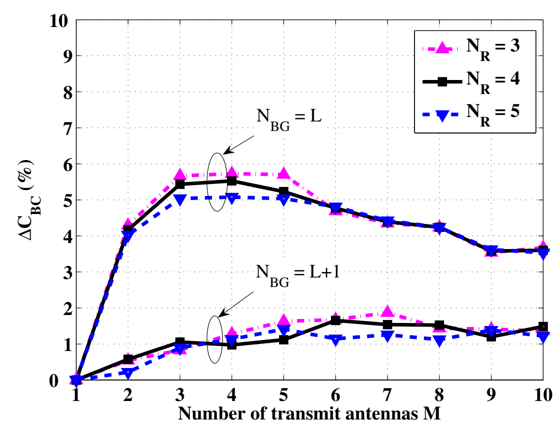

Fig. 4. Average capacity reduction with the use of BG-IDPC compared to the use of IDPC for different MIMO configurations, as a function of the number of transmit antennas M and for different values of receive antennas N_R.

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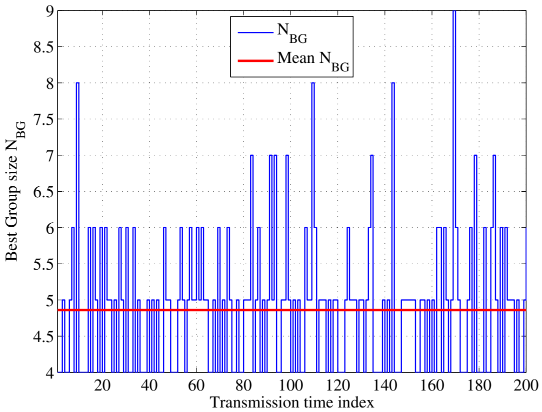

Fig. 5. Dynamic updating of the Best Group size value N_{BG} at each transmission time, in order not to exceed 5% reduction in capacity using BG-IDPC compared to that achieved using IDPC alone, with $K = 20$ and $P = 1$.

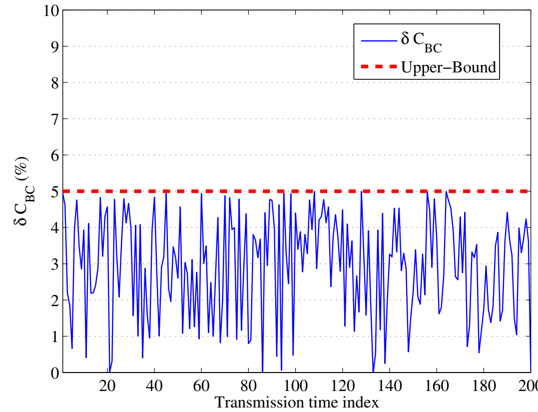

Fig. 6. Instantaneous capacity reduction with the use of BG-IDPC with dynamic updating in order not to exceed 5% capacity reduction compared to the use of IDPC, in the presence of $K = 20$ UEs and power constraint $P = 1$.

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